ON QUARK-LEPTON SYMMETRY

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Abstract

The new quantum number is introduced. It is shown that the conservation of -number results in the conservation of difference between baryon and lepton numbers. The problem of quark-lepton symmetry is discussed. It is shown that the nature of quark-lepton symmetry stems from the fact that the particles of one generation are subject to the symmetry transformation represented by 4-group of diedr.vspace5cm

The problem of quark-lepton symmetry [1]-[2] is closely related to the additive quantum number σ introduced in the work [3] with the aim to bring in some symmetry to the asymmetric disposition of upper and lower quarks along the "charge axis". The newly introduced quantum number is determined so as to result, in combination with quark electric charge q, the charge of the respective lepton. Then for u and d quarks we will have, respectively, $\sigma_u = 1/3$ and $\sigma_d = -2/3$. In general,σ -numbers for all quarks (and antiquarks) are determined by the following formula:

$$\sigma = q - 1/3, \quad \tilde{\sigma} = \tilde{q} + 1/3 \tag{1}$$

Now the quarks and leptons are positioned symmetrically along $\sigma = -q$ axis on (q, σ) plane (Fig 1.).
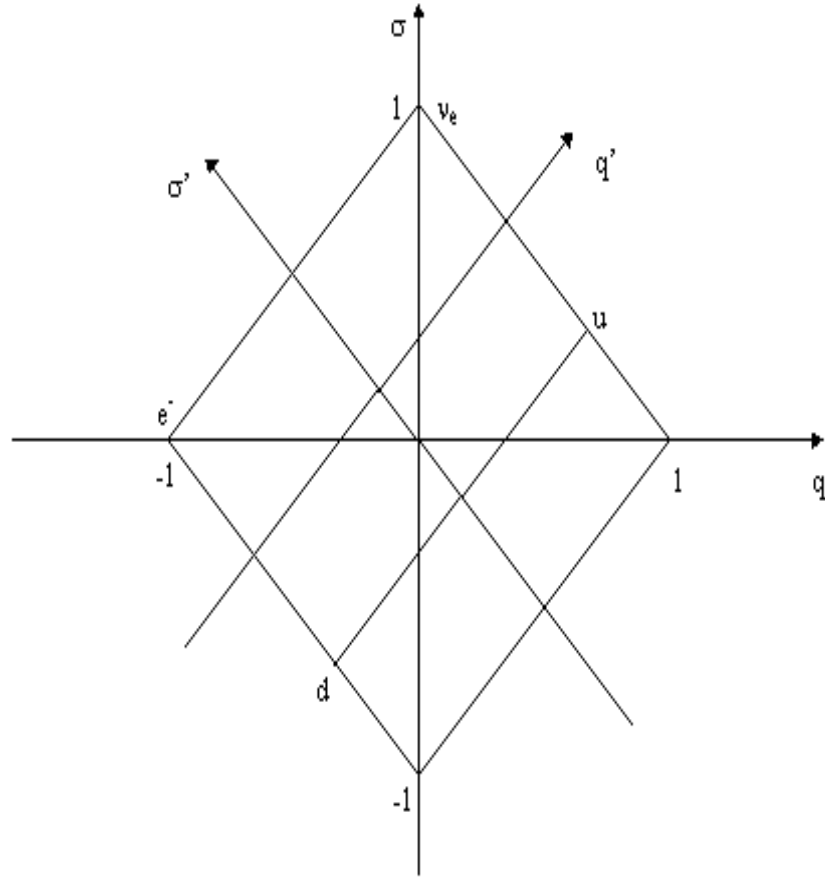
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Fig. 1

This brings about an idea that together with the law of conservation of electric charge there is realized the second law of conservation, the law of conservation of *sigma* -number. As we will see later, this idea results in

extremely valuable findings. I

In particular, we will show that the conservation of *sigma* - number results in conservation of the difference between baryon and lepton numbers obtained earlier by Georgi and Glashow from SU(5) symmetry [4]. Let us first show that σ - numbers for baryons, mesons, leptons and γ-photons are determined by the following equations:

$$\sigma_B = Q_B - 1, \quad \sigma_M = Q_M, \quad \sigma_L = Q_L + 1, \quad \sigma_\gamma = 0, \quad \tilde{\sigma} = -\sigma, \quad (2)$$

where Q is an electric charge of a particle and the sign denotes anti. Indeed, equations (2) are obvious for baryons and mesons due to their structure. They are also applicable to exotic baryons and mesons. Using (2), we can obtain, in particular,$\sigma(p) = 0, \quad \sigma(n^0) = -1, \quad \sigma(\pi^0) = 0, \quad 11\ \sigma(\wedge^0) = 1, \quad \sigma(\sum^-) = -2$, and so on. Taking into account that $\sigma(\pi^0) = 0$, we obtain $\sigma_\gamma = 0$ from $\pi^0 \Rightarrow 2\gamma$ decay. In order to find σ_l , we will give the same quantum number to all charged leptons $(\sigma_{e^-} = \sigma_{\mu^-} = \sigma_{\tau^-})$ and another quantum number to neutral leptons $(\sigma_{\nu_e} = \sigma_{\nu_\mu} = \sigma_{\nu_\tau})$. These numbers are related to each other by the formula $\sigma_{\nu_e} = 1 + \sigma_e$ derived from $n^0 \Rightarrow p + e^- + \tilde{\nu}$ reaction. Then from $uu \Leftarrow x \Rightarrow e + \tilde{d}$ reaction where the same boson may decay into antilepton + antiquark or quark pair we derive $\sigma_{e^+} = -\sigma_{e^-} = 0$, and $\sigma_\nu = 1$. As we can see, these results are in accordance with equations (2). We can rewrite them as follows:

$$\sum Q_B - N_B + \sum \tilde{Q}_B + \tilde{N}_B + \sum Q_M + \sum \tilde{Q}_m + \sum Q_L + N_L + \sum \tilde{Q}_L - \tilde{N}_L = const,$$
$$(3)$$

where N is the number of particles. Here, taking into account the conservation of electric charge, we will finally obtain the desired result:

$$(N_B - \tilde{N}_B) - (N_L - \tilde{N}_L) = const \qquad (4)$$

Now we will focus on the phenomenon of quark - lepton symmetry. Let us shift to the new system of coordinates (q', σ') with the point of origin in the center of a rectangle $e\nu_e ud$ and axes directed along the axes of symmetry (see Fig1.). The relation with an original system of coordinates is given by the following expressions:

$$q' = \frac{1}{\sqrt{2}}q + \frac{1}{\sqrt{2}}\sigma; \quad \sigma' = -\frac{1}{\sqrt{2}}q + \frac{1}{\sqrt{2}}\sigma - \frac{1}{3\sqrt{2}} \qquad (5)$$

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The coordinates of the vertices are now as follows:

$$e\left(-\frac{1}{\sqrt{2}}, \frac{\sqrt{2}}{3}\right); \quad \nu_e\left(\frac{1}{\sqrt{2}}, \frac{\sqrt{2}}{3}\right); \quad u\left(\frac{1}{\sqrt{2}}, -\frac{\sqrt{2}}{3}\right); \quad d\left(-\frac{1}{\sqrt{2}}, -\frac{\sqrt{2}}{3}\right) \quad (6)$$

The group that represents the symmetry transformation for the above mentioned rectangle, consists of the following elements:
E - identity transformation; turn around axis z for angle 2π;
A - turn around axis σ' for angle π;
B - turn around axis q' for angle π;
C - turn around axis z for angle π.
For these elements we will obtain the group multiplication table (see Table 1).

Table 1

	E	A	B	C
E	E	A	B	C
A	A	E	C	B
B	B	C	E	A
C	C	B	A	E

Under multiplication we understand the subsequent execution of the corresponding operations. Besides that, all the elements of the group have order 2 (except for identity element E), since $\chi^2 = E$ and $\chi^{-1} = \chi$, where χ is an arbitrary element of the group. Thus, the totality of the elements E, A, B, C makes up the Abelian group. The similitude transformation of a regular polygon is expressed by means of the following matrices [5]

$$D_k = \begin{pmatrix} \cos\frac{2\pi k}{n}, & \sin\frac{2\pi k}{n} \\ -\sin\frac{2\pi k}{n}, & \cos\frac{2\pi k}{n} \end{pmatrix} \quad U_k = \begin{pmatrix} -\cos\frac{2\pi k}{n}, & \sin\frac{2\pi k}{n} \\ \sin\frac{2\pi k}{n}, & \cos\frac{2\pi k}{n} \end{pmatrix} \quad (7)$$

where k=0,1,2,, n-1. These 2n-dimensional matrices make up the group oforder 2n known as the group of diedr. In case of n=2 we have the simplest

case of the group with elements

$$E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \quad A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}; \quad B = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}; \quad C = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix};$$

(8)

Taking into account that the group (8) is isomorphic to the above mentioned group, we come to conclusion that (8) is the matrix representation of the group. Thus, if particles of the same generation are located in the vertices of rectangle (6), on the pla ne (q', σ'), this distribution is subject to the symmetry transformation described by the group (8). It is also easy to show that if distribution of particles on the plane (q', σ') is subject to the symmetry transformation described by the group (8) and if coordina tes of any arbitrary particle from (6) coincide with one of the vertices, there should be three more particles (and only three, without taking into account the color of the quarks) whose coordinates coincide with the remaining vertices.

In conclusion, we can say that the nature of the quark-lepton symmetry could be explained by -symmetry, which, in its turn, is caused by newly introduced in the work [1] quantum number σ . In other words, the quark-lepton symmetry stems from the fact tha t particles of the same generation are subject to the symmetry transformation represented by 4-group of diedr.

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